

The transverse breathing mode of an elongated Bose-Einstein condensate

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We study experimentally the transverse monopole mode of an elongated rubidium condensate. Due to the scaling invariance of the non-linear Schrödinger (Gross-Pitaevski) equation, the oscillation is monochromatic and sinusoidal at short times, even under strong excitation. For ultra-low temperatures, the quality factor $Q = \omega_0/\gamma_0$ can exceed 2000, where ω_0 and γ_0 are the mode angular frequency and damping rate. This value is much larger than any previously reported for other eigenmodes of a condensate. We also present the temperature variation of ω_0 and γ_0 .

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Collective excitations constitute one of the main sources of information for understanding the physics of many-body systems. Since the discovery of Bose-Einstein condensation (BEC) in weakly interacting bosonic gases [1] much theoretical and experimental effort has been devoted to the study of the low energy excitations of these systems [2, 3, 4, 5, 6]. At ultra-low temperatures, the measured frequencies of these excitations are found to be in very good agreement with the ones derived from a simple linearization of the Gross-Pitaevskii equation, which describes the evolution of the condensate wave function.

The temperature dependence of the damping and frequency of these oscillations has attracted strong attention of both experimental [7, 8, 9] and theoretical groups [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Indeed, a dilute BEC is a very convenient system for the study of dissipation in a macroscopic quantum object. At present, there still exist significant discrepancies between some available experimental results and the corresponding theoretical predictions (see e.g. [15, 20]). Therefore new measurements on simple and well identified modes are clearly needed to test the various theoretical approaches.

It has been known for some time that in an isotropic trap, the monopole (or breathing) oscillation mode may exhibit very unusual features. For instance, Boltzmann noticed that, within the framework of the equation which bears his name, the breathing mode of a set of particles confined in a 3D isotropic harmonic trap is undamped, irrespective of the ratio between the average collision rate and the trap frequency [21, 22, 23]. More recently it was pointed out that in a 2D isotropic harmonic trap, the breathing mode of identical bosons interacting via a contact potential is also undamped [24, 25]. This property, which holds at any temperature, can be related to the existence of a hidden symmetry of the problem described by the two-dimension Lorentz group $SO(2,1)$ [25].

The purpose of the present work is to investigate experimentally the transverse breathing mode of an elongated condensate. We find that this mode exhibits very unique features, such as an extremely large quality factor $Q = \omega_0/\gamma_0 \sim 2000$ for temperatures much lower than the condensation temperature. Here ω_0 and γ_0 are the angu-

lar frequency and damping rate, respectively. This quality factor, much larger than any previously reported value for an oscillation mode of a gaseous BEC, should provide a sensitive test of the various theoretical models of dissipation at ultra-low temperature. Another very unusual property lies in the fact that the measured frequency ω_0 is nearly independent of temperature. Furthermore, we find that the transverse area of the condensate undergoes a pure, isochronal oscillation even for strong excitation. This striking behavior emerges from a scaling invariance of the time-dependent Gross-Pitaevski equation.

Our ^{87}Rb condensate is formed by radio-frequency (rf) evaporation of 10^9 atoms in a Ioffe-Pritchard magnetic trap. The initial temperature of the cloud pre-cooled using optical molasses is $100\text{ }\mu\text{K}$. We vary the final temperature of the gas and the number of atoms in the condensate by adjusting the final rf, ν_f . The condensation threshold is reached at $T_c = 290\text{ nK}$, with $N_c = 8.4 \cdot 10^5$ atoms [26]. The coldest sample reliably reproducible ($T \sim 40\text{ nK}$) is a quasi-pure condensate with $N_0 = 10^5$ atoms. It is obtained using $\nu_f = \nu_0 + 6\text{ kHz}$, where ν_0 is the rf at which the trap is emptied.

The trap has a longitudinal frequency $\omega_z/(2\pi) = 11.8(1)\text{ Hz}$ for the atoms prepared in the stretched state $m = 2$ of the $5S_{1/2}$, $F = 2$ ground state. The transverse magnetic gradient is $b'_\perp = 1.33\text{ T m}^{-1}$ and the transverse frequency ω_\perp is adjusted by varying the bias field B_0 at the center of the trap. We choose $B_0 = 0.844 \cdot 10^{-4}\text{ T}$, which leads to $\omega_\perp/(2\pi) = 182.6(4)\text{ Hz}$ [27]. Both frequencies ω_z and ω_\perp are measured by monitoring the oscillation of the center-of-mass of the atom cloud.

Once the condensate is formed, we excite the transverse breathing mode by modifying the bias field from B_0 to B'_0 , which changes the frequency ω_\perp to $\omega'_\perp > \omega_\perp$. After a time $\tau \ll 2\pi/\omega'_\perp$, we set the bias field back to its original value. We let the cloud oscillate freely in the magnetic trap for an adjustable time t and then measure the transverse density profile of the condensate after a period of free expansion. In this pursuit, we suddenly switch off the magnetic field, allow for a 25 ms free-fall, and image the absorption of a resonant laser by the expanded cloud. The imaging beam propagates along the z

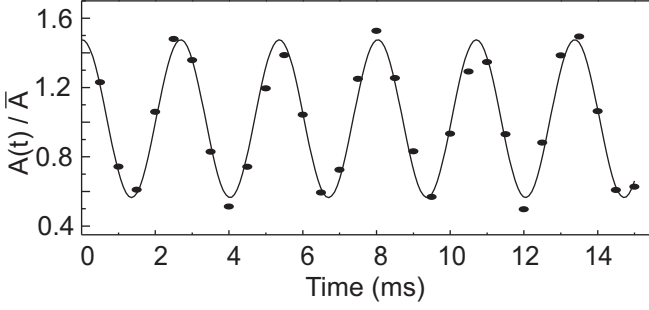


FIG. 1: Variation of $A(t) = R_x^2(t) + R_y^2(t)$, after a strong excitation of the transverse breathing mode. The line is a sinusoidal fit of the data ($\omega_\perp/2\pi = 250$ Hz, $\tau = 800$ μ s).

axis. We fit the density profile of the condensate assuming the parabolic shape of the Thomas-Fermi distribution and extract the radii R_x and R_y in the plane $z = 0$. The temperature is obtained from a Gaussian fit of the uncondensed part of the distribution.

Within the Thomas-Fermi approximation, the evolution of the trapped condensate after the excitation of the transverse breathing mode is well described by a time-dependent scaling transformation [24, 28, 29]. The spatial density $\rho(\mathbf{r}, t)$ at time t is deduced from the spatial density at time 0 (i.e. just after the transverse frequency is set back to its value ω_\perp) by:

$$\rho(x, y, z, t) = \frac{1}{\lambda_\perp^2 \lambda_z} \rho\left(\frac{x}{\lambda_\perp}, \frac{y}{\lambda_\perp}, \frac{z}{\lambda_z}, 0\right).$$

The evolution of the scaling parameters λ_\perp and λ_z is given by [28, 29]:

$$\ddot{\lambda}_\perp = \frac{\omega_\perp^2}{\lambda_\perp^3} - \omega_\perp^2 \lambda_\perp \quad \ddot{\lambda}_z = \frac{\omega_z^2}{\lambda_\perp^2 \lambda_z^2} - \omega_z^2 \lambda_z. \quad (1)$$

We start our analysis with the evolution for relatively short times $t \ll 2\pi/\omega_z$. In this case one can neglect the variation of λ_z and integrate the equation for λ_\perp :

$$\lambda_\perp(t) = (\cosh(\alpha) + \sinh(\alpha) \sin(2\omega_\perp t - \beta))^{1/2},$$

where the parameters α and β depend on the initial conditions. One remarkable consequence of this solution is that the quantity $A(t) = R_x^2(t) + R_y^2(t) = \lambda_\perp^2(t) A(0)$ always undergoes a sinusoidal oscillation at frequency $2\omega_\perp$, irrespective of the strength of the excitation [30]. This is confirmed by the results of fig. 1, where we observe a sinusoidal oscillation of $A(t)$ with a factor 3 between the maxima and the minima. The frequency of the oscillation is indeed found to be equal to $2\omega_\perp$, within our experimental uncertainty (± 0.5 %).

At longer times one has, in principle, to take into account the variation of λ_z . We restrict our analysis to small amplitude oscillations and write $\lambda_i = 1 + \epsilon_i$, where $\epsilon_\perp, \epsilon_z \ll 1$. The linearized equations of motion

$$\ddot{\epsilon}_\perp + \omega_\perp^2 (4\epsilon_\perp + \epsilon_z) = 0 \quad \ddot{\epsilon}_z + \omega_z^2 (2\epsilon_\perp + 3\epsilon_z) = 0$$

give rise to two modes [2, 4]. One is a fast mode of frequency $\sim 2\omega_\perp$ corresponding to $\epsilon_\perp/\epsilon_z = \omega_\perp/\omega_z$. The other one is a slow mode of frequency $\sqrt{5/2} \omega_z$ with $\epsilon_\perp/\epsilon_z = -1/4$. Our excitation scheme corresponds to $\dot{\epsilon}_\perp \neq 0$ at time 0 while the three other quantities ϵ_\perp , ϵ_z and $\dot{\epsilon}_z$ are approximately zero at initial time. Under these conditions, we mainly excite the fast mode $2\omega_\perp$ since the relative weight of the slow mode in the evolution of λ_\perp is only $\omega_z/(\sqrt{40}\omega_\perp) < 0.01$. For this fast mode, the condensate dynamics consists essentially in a transverse monopole oscillation since $\epsilon_z \ll \epsilon_\perp$.

We investigate the oscillation and the damping of this mode on a long time scale (up to 0.7 s). An example is given in fig. 2 for $T = 40$ (± 20) nK (note that for such cold clouds, corresponding to a quasi-pure condensate, the temperature can only be inferred from the final rf used for the evaporation). The evolution of $A(t)$ is well fitted by a damped sinusoidal function, $A_0 + \delta A_0 \cos(\omega_0 t + \phi) e^{-\gamma_0 t}$, with $\delta A_0/A_0 = 0.063$ (4), $\omega_0/2\pi = 366.3$ (5) Hz and $\gamma_0 = 1.2$ (2) s^{-1} . This corresponds to a quality factor $Q = \omega_0/\gamma_0 \sim 2000$ much larger than any previously reported for other eigenmodes of a BEC. For instance, measurements performed with a TOP trap ($\omega_z = 2\sqrt{2}\omega_\perp$) gave $Q \sim 200$ for the lowest $m = 0$ mode [7]. For a cigar shape trap, measurements of the low frequency mode discussed above ($\omega = \sqrt{5/2}\omega_z$) led to $Q \simeq 80$ [8]. A smaller value ($Q \sim 25$) was measured for the scissors mode [9]. The present quality factor is also one order of magnitude larger than what we find for the transverse quadrupole mode under the same experimental conditions [31].

We measure the oscillation frequency and the damping rate over a large temperature range compatible with the detection of a condensate. The results are given in fig. 3, together with the total number of atoms N . The measurement of γ_0 is made from the decay of the oscillation amplitude between 0 and 200 ms. For the points of lowest and highest temperature in figs. 3b and 3c, we calculate, including mean field and finite size corrections [10], $T/T_c = 0.3$ and 0.8, respectively.

The frequency ω_0 is found to be very close to the Thomas-Fermi prediction $2\omega_\perp$, and it varies only slightly with temperature (fig. 3b). In particular, we do not observe the 10 % relative decrease predicted in [14] when T/T_c increases from 0 to 0.8.

The damping rate γ_0 varies approximately linearly with temperature in the range 50 – 200 nK. The linear variation of γ_0 for $k_B T$ larger than $\hbar\omega_0$ (~ 20 nK) and than the chemical potential μ (~ 60 nK for the coldest point in fig. 3) is expected if one assumes either Beliaev-type or Landau-type damping of the monopole excitation (see e.g. [32]). In the Beliaev mechanism, the monopole excitation decays into two excitations with a lower energy. For Landau damping, it annihilates in a collision with a thermal excitation to give rise to another excitation. The calculation of the corresponding damping

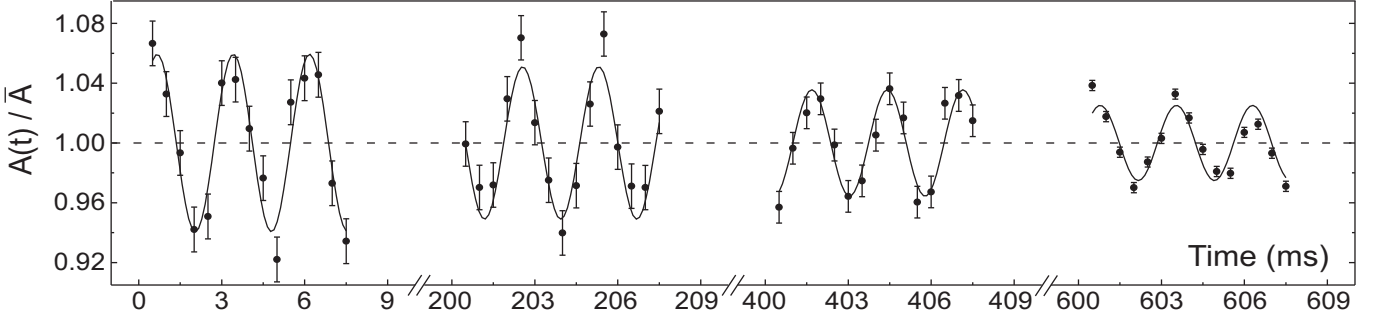


FIG. 2: Long time evolution of $A(t)/\bar{A}$ at low temperature (40 nK), where \bar{A} is the average of $A(t)$ over each time interval [0,9 ms], [200 ms,209 ms] ... The oscillation is well fitted by a damped sinusoid. Here $\omega'_\perp/2\pi = 230$ Hz and $\tau = 75 \mu\text{s}$.

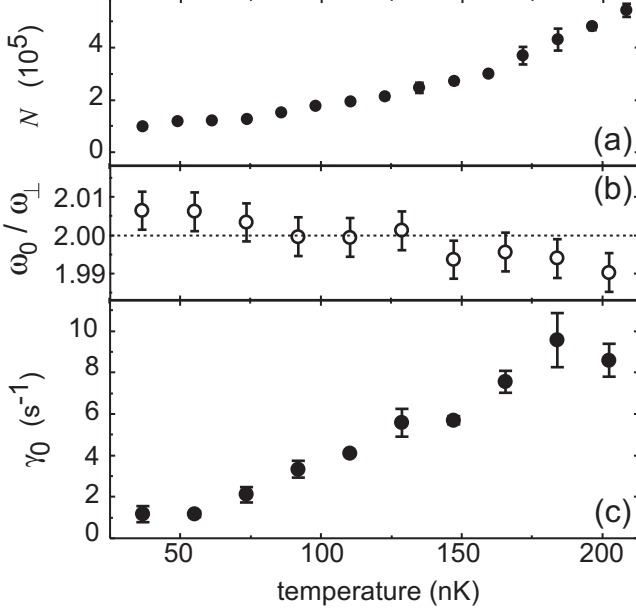


FIG. 3: (a) Total number of atoms N , (b) breathing mode frequency ω_0/ω_\perp and (c) damping rate γ_0 , as a function of temperature. Each point in (b) and (c) corresponds to $\nu_t - \nu_0 = 6$ kHz, 9 kHz, ..., 33 kHz ($\omega'_\perp/2\pi = 230$ Hz, $\tau = 150 \mu\text{s}$).

rate is quite complex for inhomogeneous systems and can lead to drastically different results for modes with similar energies, depending on the stochastic or regular nature of the relevant thermal excitations [32]. In this respect it is worth noting that the frequency of this transverse breathing mode is an integer multiple of the trap oscillation frequency, which is usually not the case for other modes of the condensate.

For a very low temperature ($T \sim 40$ nK) and an increasing excitation strength, we observe a significant deviation of the long term behavior of $A(t)$ with respect to the simple damped sinusoid shown above. Fig. 4 shows the oscillation amplitude for the weak excitation (\bullet , $\tau = 75 \mu\text{s}$) together with two sets of data obtained for $\tau_1 = 300 \mu\text{s}$ (\square) and $\tau_2 = 450 \mu\text{s}$ (\blacktriangle). One clearly sees that for strong excitation the amplitude of the oscillations passes through a local minimum around $t = 300$ ms before increasing again, and decaying finally after 700 ms. We fit our experimental data with the phenomenological

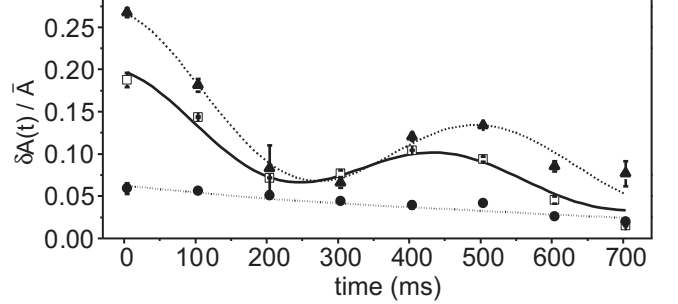


FIG. 4: Long term evolution of the amplitude of the breathing mode for different excitation strengths (\bullet : $\tau = 75 \mu\text{s}$, \square : $\tau_1 = 300 \mu\text{s}$, \blacktriangle : $\tau_2 = 450 \mu\text{s}$). Each point at $t = 0, 100$ ms, ..., 700 ms represents the amplitude of the breathing mode measured during the period $[t, t + 8$ ms]. The fit function is indicated in the text. Here $\omega'_\perp/2\pi = 230$ Hz.

function $A(t) = A_0 + \delta A(t) \cos(\omega_0 t + \phi)$ where the amplitude $\delta A(t)$ is now slowly modulated in time, in addition to the exponential decay already discussed: $\delta A(t) = \delta A_0 (1 - \xi \sin^2(\delta\omega t)) e^{-\gamma t}$. The best fit parameters are for these two cases $\gamma_1 = 1.48$ (15) s^{-1} , $\delta\omega_1/(2\pi) = 1.08$ (4) Hz, $\xi_1 = 0.5$ (1), and $\gamma_2 = 1.35$ (15) s^{-1} , $\delta\omega_2/(2\pi) = 0.96$ (4) Hz, $\xi_2 = 0.6$ (1).

Two scenarios can be invoked to explain this evolution of $A(t)$, which is not predicted by (1). A first explanation could consist in assuming that we actually excite two modes whose frequencies $\sim \omega_0$ differ by a small amount and that we observe the beating between these modes. However, if we assume that these two modes are excited linearly, the relative amplitude of the beating should not depend on the excitation strength, contrary to the observed. Furthermore, this explanation is not confirmed by a linear analysis of the time dependent Gross-Pitaevskii equation for our experimental conditions [33]. A second, more plausible, scenario involves a non-linear process, in which the monopole excitation may be converted in a reversible way into a sum of other excitations of frequency ω_j . This process is efficient if it is resonant: $\omega_0 \simeq \sum_j \omega_j$. In this context, the modulation in fig. 4 can be interpreted as a parametric oscillation.

As a possible candidate for such a parametric (or Belevskii type) oscillation [34], one may consider the excitation of the pair of modes $l = 2, m = \pm 1$ [2, 6, 35]. The

frequency of each mode is $(\omega_{\perp}^2 + \omega_z^2)^{1/2}$ so that the resonance condition given above is indeed nearly fulfilled. By imaging the condensate along the y direction, we observe a weak excitation of this mode. However, no increase in its amplitude is found around $t=300$ ms when the amplitude of the breathing mode collapses (cf. fig. 4).

Alternatively one may consider the excitations consisting in a pair of phonon-like excitations propagating along z with momentum k and $-k$, each with an energy $\hbar\omega_0/2 \sim \hbar\omega_{\perp}$ [17]. To test this latter hypothesis, we perform the same experiment for various values of ω_{\perp} , so that it spans the whole interval between two successive phonon modes (typically 6 Hz in this energy domain). However, expecting a resonance when $\omega_0/2$ coincides with the frequency of a phonon mode, we do not observe any significant change in the evolution of $A(t)$.

In any case we note that for the long oscillation times involved here, small non-linear terms in the confining potential (varying as x^2z and y^2z) may play a significant role. Further work investigating this possibility is under progress in our laboratory.

In summary we have presented in this letter a study of the transverse breathing mode of an elongated Bose-Einstein condensate. For low temperatures and short evolution time, the observed isochronal and sinusoidal oscillation is in excellent agreement with the prediction based upon the scaling invariance of the Gross-Pitaevski equation. However, the temperature dependence of the frequency and damping of the oscillation reveals, on the contrary, some unexpected features. First, the observed damping is comparatively ten times smaller than what was previously reported for other modes of a BEC (quality factor of 2000 instead of 100-200). Second, the frequency of the breathing mode is found to be quasi-independent of temperature, in contradiction with recent theoretical predictions. Finally the long time evolution of this mode when strongly excited reveals an unexpected non-linear mixing phenomenon. This could be explored further using the formalism of [36], by looking at a possible dynamical instability of the Gross-Pitaevskii equation in this regime of strong excitation.

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